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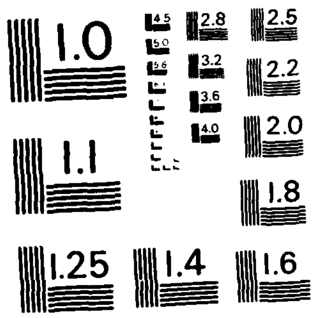

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10 May 1983

The Fast Fourier Transform as a Signal Processing Technique

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PREFACE

This document was prepared under NUSC Project No. A99934, principal investigator-- C.A. Ledoux (Code 3631).

The author wishes to thank T.A. Galib, Jr. (of NUSC Code 3634) and W. Boober (of Vitro Laboratories), who reviewed the original manuscript and offered many helpful suggestions. The author is also grateful to his supervisors (J.E. Sirmalis, J.R. Short, and B.J. Myers) without whose support this document could not have been written. Finally, of the many sources from which information was extracted in preparing this document, a particularly helpful guide has been "FFT Fundamentals and Concepts" written by R. Ramirez of Tektronix Inc.

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J.E. Sirmalis
Head, Weapon Systems Department

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The proliferation of digital equipment for use in making all types of measurements has necessitated drastic changes in the signal detection techniques used to quantify measured signals. More specifically, there has been an increased use of the kinds of digitizers whose output consists of sequential time slices of the measured signal in binary coded values that correspond to amplitude and time. This type of information is directly transferable to a computer memory, where it

20. ABSTRACT (Cont'd)

can be processed by a wide variety of digital techniques. One of the most useful digital processes used to resolve amplitude and phase spectra from a digitized time-domain signal is the fast Fourier transform (FFT). This document attempts to give the reader a sound understanding of what the FFT does and how it can be applied to various measurement needs.

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INTRODUCTION

BACKGROUND

The Fourier series and integral were developed during the early 1800s by a French mathematician and physicist, Jean Fourier, originally for the purpose of evaluating heat transfers. Later, the Fourier series became known as a method of extrapolating the frequency spectrum of any time-related waveform. However, the calculations required to perform the conversion of time domain to frequency domain were so laborious and time consuming that the use of the Fourier series was not practical.

With the advent of computers, digital processing techniques were developed to compute the Fourier series. These techniques became known as the discrete Fourier transform or DFT. Further developments of DFT methods, which were more efficient than the original DFT, became known as the fast Fourier transform or FFT.

PURPOSE

This document is intended to familiarize the reader with all aspects of the FFT important to its use as a signal processing technique. Although this subject is explained in numerous publications, the treatment is usually in esoteric terms and symbols more appropriate to the world of the mathematical theorist than to that of the inexperienced technician who wants to make practical use of the FFT.

Having expended much effort in resolving the intricacies of this subject, the author hopes that the practical treatment presented in this document will help others save time, expense, and frustration when seeking a basic understanding of the fast Fourier transform.

SCOPE

Since it is assumed that the computer's operating system has some type of FFT statement that automatically performs the necessary calculations, a technical explanation of the mathematics involved is not required for the practical approach presented in this report. Such information is available elsewhere. After a brief overview of the more general detection techniques,

a step-by-step description is given showing what happens to a time-domain waveform as it is converted by FFT calculations. Guidelines are provided for using the FFT, and common pitfalls encountered in applying the technique are discussed, together with suggestions for circumventing such problems.

The potential usefulness of this document to various readers may be assessed on the basis of their recognition and understanding of the following example of an efficient DFT formula:*

For the Kth spectral line of the frequency spectrum,

$$F_K = \frac{1}{N} \sum_{N=0}^{N-1} fn \exp(-j2\pi nK/N) = C_K - js_K ,$$

where N is the number of digitized points; C_K and $-S_K$ are the real and imaginary parts of line F_K; fn is the frequency spectrum; and K is the number of cycles of the test sinusoid.

Those readers who recognize and understand this formula will not benefit significantly from the contents of this document. Those readers who do not recognize or understand the formula should find the step-by-step explanations of the DFT and FFT methods given in this document of value.

*From James D. Gray, "Amppha-RMS Amplitude, Phase and Percent Harmonic Distortion of the Kth Spectral Line of the DFT of a Real Sequence," USRD Tecnical Note No. 4, Naval Research Laboratory, Underwater Sound Reference Division, Orlando, FL, 25 December 1981.

OVERVIEW OF SIGNAL DETECTION TECHNIQUES

This section gives a brief overview of the available detection techniques that are used in the absence of digitizers. It is provided to give a perspective on the importance of the FFT.

PEAK DETECTION

Peak detection is the simplest method used to quantify a voltage level. For direct current (dc) voltage, the current (I) is caused to flow through a meter by the application of the voltage (E) to be measured (figure 1).

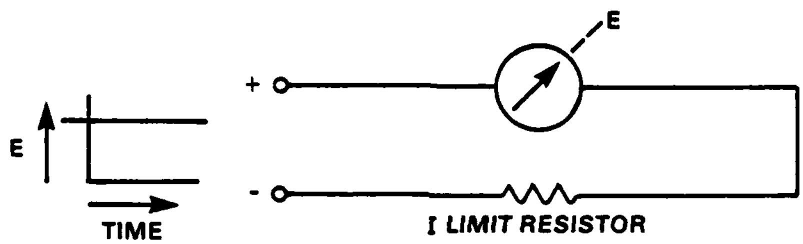

Figure 1. Direct Current Voltmeter

The wide usage of alternating current (ac) voltages caused a rectifier to be added to the circuit and rendered it an ac voltmeter (figure 2). This circuit is known as a "peak detector" because the meter output is proportional to one-half of the peak-to-peak (maximum) value of the applied voltage.

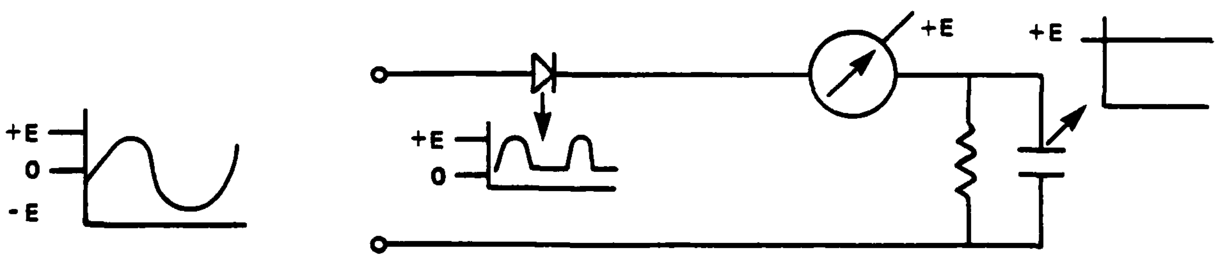

Figure 2. Alternating Current Voltmeter

WAVEFORM RELATIONSHIPS

At this point, it is useful to recall some of the more important relationships of an ac waveform. In figure 3, a perfect sinusoid (sine wave) is assumed. The root mean square (rms) is the equivalent dc value of the sine wave:

$$\text{rms} = 0.707 \times \text{peak} = \frac{(P - P)}{2.828} = \frac{P}{1.414},$$

$$P = 1/2 \times (P - P).$$

In relationships for waveforms other than sine waves, the true rms value is obtained by rms evaluation of an adequate number of points on the waveform being measured.

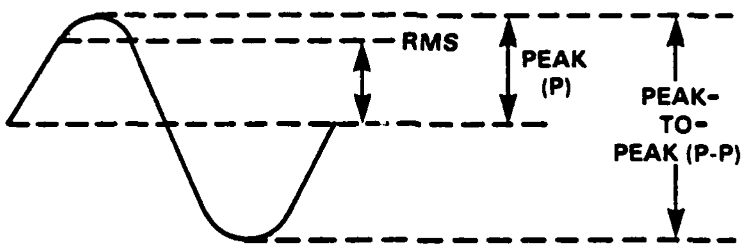

Figure 3. Waveform Relationships

TRUE RMS DETECTION

The circuit of figure 2 can be calibrated to read rms values by the proper choice of current-limiting resistors, but the values would be accurate only for sine waves. This is not a satisfactory situation for real-world applications.

There are some analog solutions to this problem. An older one is the electrodynamometer movement, and a more recent one is the Hall effect cell. These analog devices respond essentially to the dc value of any waveform.

SAMPLE AND HOLD DETECTION

There are many instances when a signal is not of a continuous nature but occurs in the form of a burst, as shown in figure 4. The circuits previously shown would not work for such an applied signal. Therefore, the various detectors are fitted with circuits that allow the detectors to respond to the applied signal at selected intervals called "gates," and to "hold" the measured values until they are cleared. The gating (sample and hold) process is usually synchronized with the [illegible]its of the measured signal.

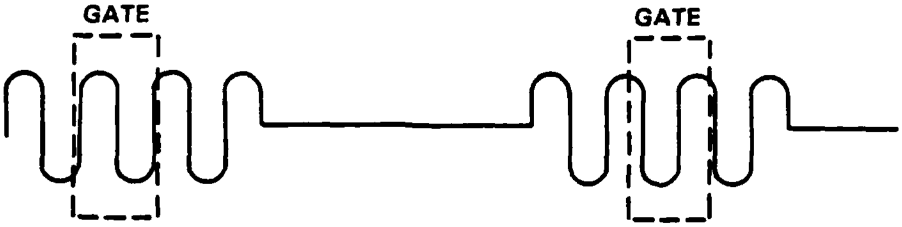

Figure 4. Sample and Hold Circuit

INFORMATION AVAILABLE FROM VARIOUS SAMPLED WAVEFORMS

Sine Waves

As shown in figure 5, amplitude information is derived from the P-P value or true rms detection. Relative phase information can be derived from TX or any other point desired. Frequency can be derived from the formula F = 1/TY.

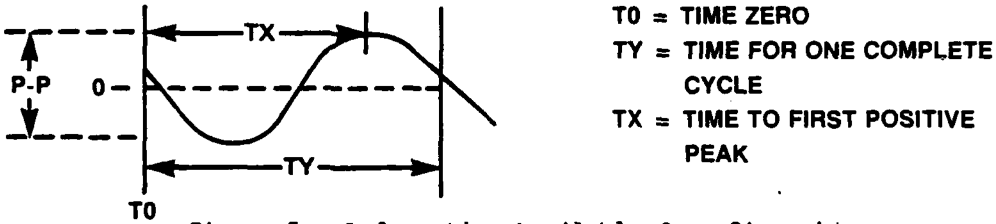

Figure 5. Information Available from Sinusoids

Envelope Signals

In figure 6, the "envelopes" (+ and -) are the maximum and minimum values of sine waves that make up a burst. The envelope values give the following information:

1. The maximum P-P value during the time window.

2. The energy contained in the time window, which can be derived by summing the squares of an adequate number of points on the envelope.

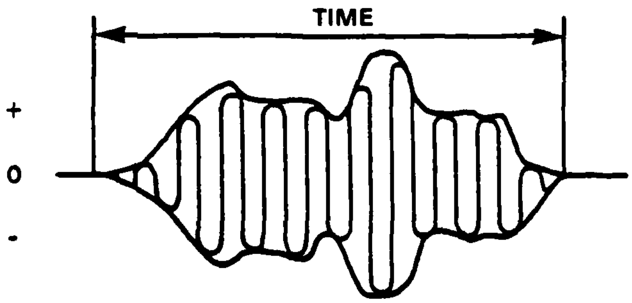

Figure 6. Envelope of Burst Signal

Pulse Signals

The signal in figure 7 is a combination of sine waves of various amplitudes, frequencies, and phases. As shown later, individual sine waves and their components can be resolved using FFT techniques.

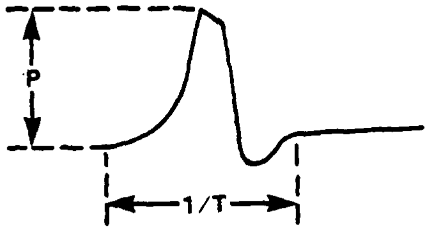

Figure 7. Pulse or Shock Signal

USING A DIGITIZER FOR SIGNAL ACQUISITION

Capturing of the signals typified in figures 5, 6, and 7 is best done by using a digitizer.

Figure 8 shows that, with a proper choice of vertical scale (Y) and horizontal (X) sampling frequency, any waveform that is applied, within the usable working range of the digitizer, will be converted to a sequence of binary codes that can be processed by a computer. The computations can yield any type of signal processing technique desired: P, P-P, true rms, envelope, and energy or frequency spectrum extrapolation using the FFT.

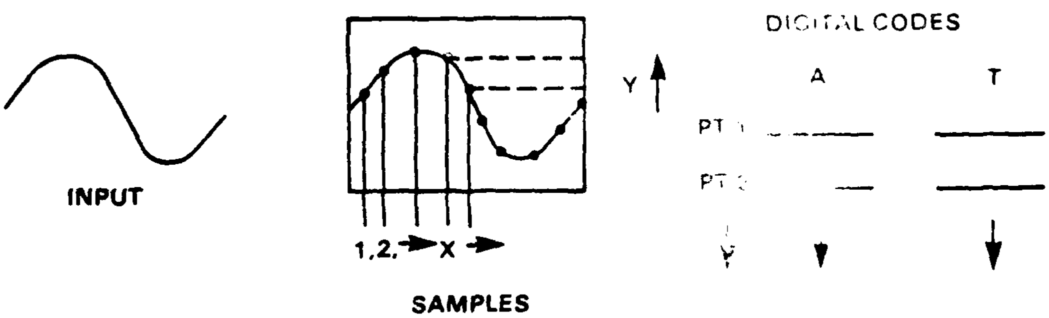

Figure 8. Digitizer Acquisition

BASIC DESCRIPTION OF THE FOURIER METHOD

CONVERSION OF TIME TO FREQUENCY USING THE DFT

The DFT converts a time-domain waveform, from the output of a digitizer as shown in figure 8, to a frequency-domain spectrum as shown in figure 9. The domain conversion is done by a mathematical treatment of sequential time slices. The data sequence output of the DFT is interleaved as follows:

1	F(1)	Real part (e.g., dc value of signal)
	F(2)	Real part of value of DFT at Nyquist frequency (at one-half the sampling frequency)
2	F(3)	Real part of first Fourier coefficient
	F(4)	Imaginary part of first Fourier coefficient
3	F(5)	Real part of second Fourier coefficient
	F(6)	Imaginary part of second Fourier coefficient
↓	F(7)	
recurring ↓		
N/2	F(N-1)	Real part of N/2-1 Fourier coefficient (Nyquist)
	F(N)	Imaginary part of N/2-1 Fourier coefficient

The time domain size must be a power of 2. The real and imaginary parts are interleaved. The sign of the outputs (+ or -) indicates the phase.

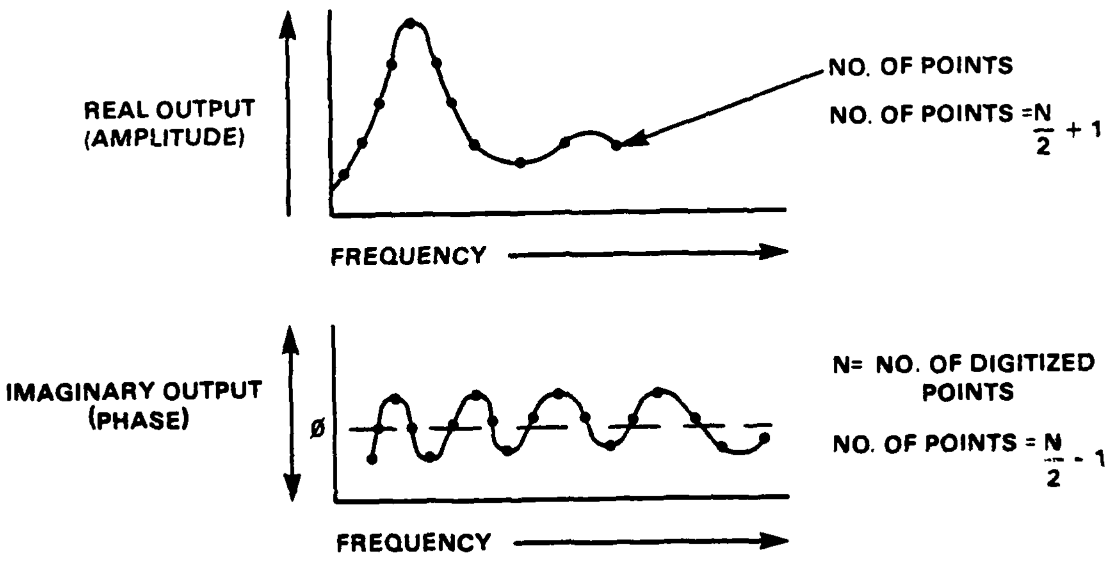

Figure 9. Frequency-Domain Output of FFT

ADVANTAGES OF THE FFT METHOD

The DFT requires N^2 major operations to resolve time to spectra. Since the FFT algorithm reduces the number of major operations to $N \log_2 N$, a significant time saving, the FFT has become the most popular and efficient method of performing the conversion.

CONSIDERATIONS FOR USE OF FFT

FFT OUTPUTS

Frequency Spectrum Output of the FFT

The waveform shown in figure 10 is the output of a 512-point digitizer (time domain). The sampling frequency (SF) is:

$$SF = \frac{\text{No. of Points}}{\text{Total Time (T)}} = \frac{512}{10 \times 10^{-6}} = 51.2 \text{ MHz.}$$

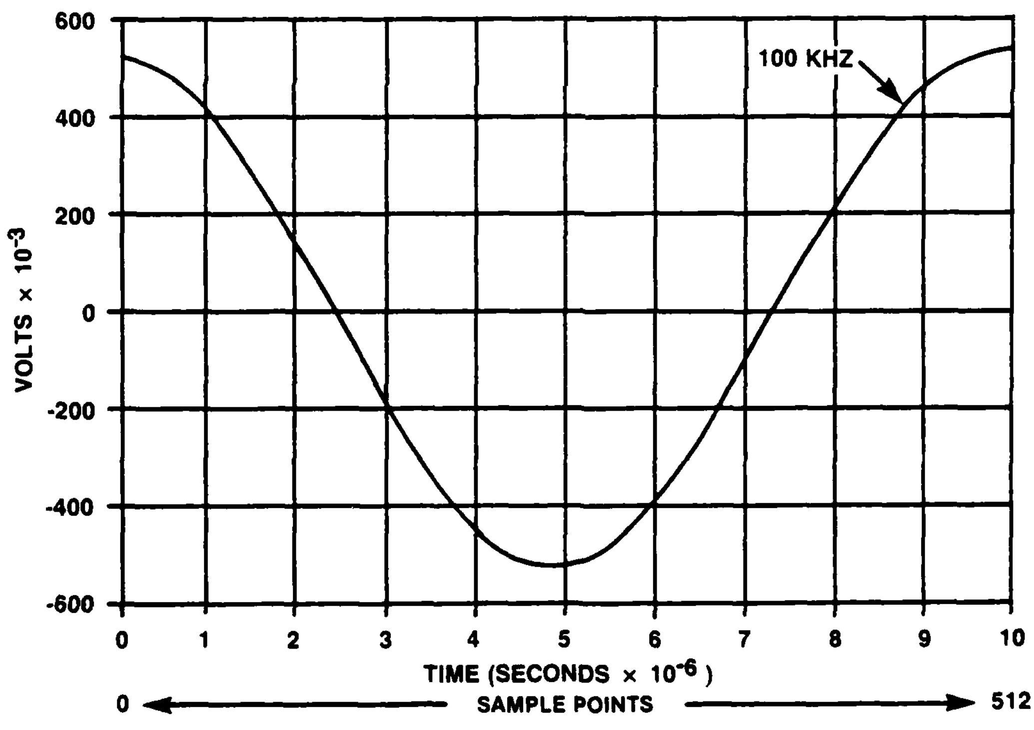

Figure 10. Time-Domain Waveform

The FFT outputs the frequency-domain amplitude and phase (real and imaginary numbers) as shown in figure 11. The real numbers are cosine related and the imaginary numbers are sine related. The maximum frequency component is 25.6 MHz or

$$\text{Maximum Frequency} = 1/2 \text{ SF}.$$

This is known as the Nyquist frequency and will be discussed later.

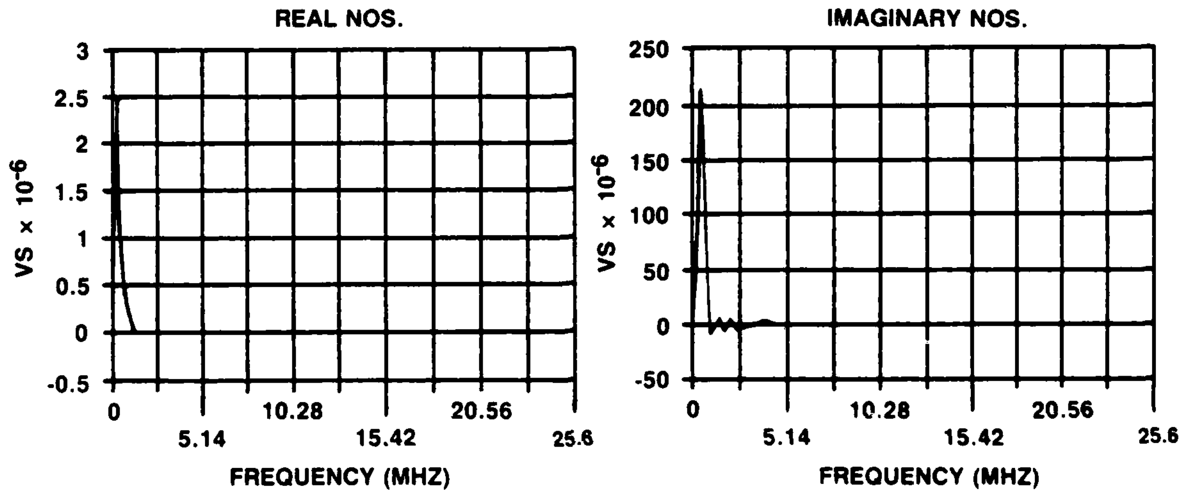

Figure 11. FFT Output

An enlargement of the first 350 kHz of the amplitude (real numbers) graph is shown in figure 12, where the output points have been joined for clarity. The frequencies output are at 100 kHz increments — 0, 100, 200, 300 kHz, etc. The frequency increment (FI) is:

$$\text{FI} = \frac{\text{Sampling Frequency}}{\text{No. of Points}}$$

$$= \frac{\text{Sampling Frequency}}{\text{No. of Digitized Points}}$$

$$= \frac{51.2 \times 10^{-6}}{512} = 100 \text{ kHz}.$$

There are 257 frequency points output (including zero frequency). The number of actual frequencies is:

$$\frac{\text{No. of Digitized Points}}{2} = \text{No. of Spectral Lines}.$$

The phase information (imaginary numbers) occurs at the same spectral points as the amplitude information (real numbers) occurs, as shown in figure 13. Note that the vertical scale units are specified as VS (volt second) units, which are the normal outputs of the DFT. The VS units can be converted to familiar units of volts and radians by a process known as "polar conversion."

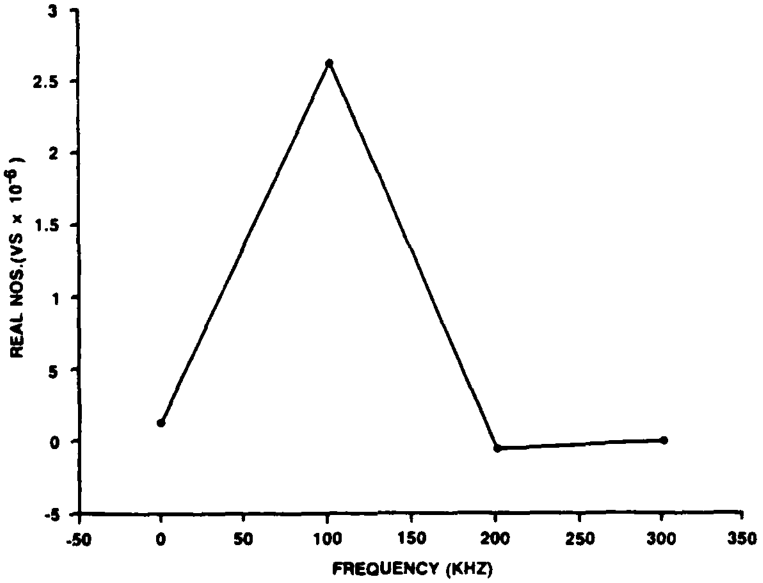

Figure 12. Expansion of First Four FFT Real Output Points

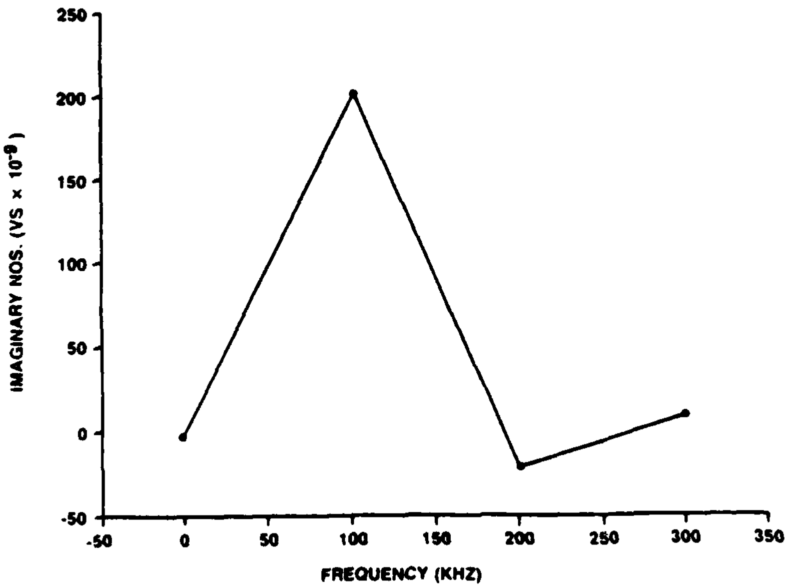

Figure 13. Expansion of First Four Imaginary Output Points

Polar Conversion

The FFT outputs are converted to volts and radians with the use of algorithms contained in the POLAR command. The technical aspects of polar conversion are found in various books on signal processing. A simplified explanation follows.

Given the real (re) and imaginary (im) outputs of the FFT, the following equations perform the conversion to polar coordinates:

$$|\text{Magnitude}| = \sqrt{(re)^2 + (im)^2} \ ,$$

$$\text{Phase} = \tan^{-1}(im/re) \ .$$

The polar outputs occur at the same spectral lines as the FFT outputs, as shown in figure 14.

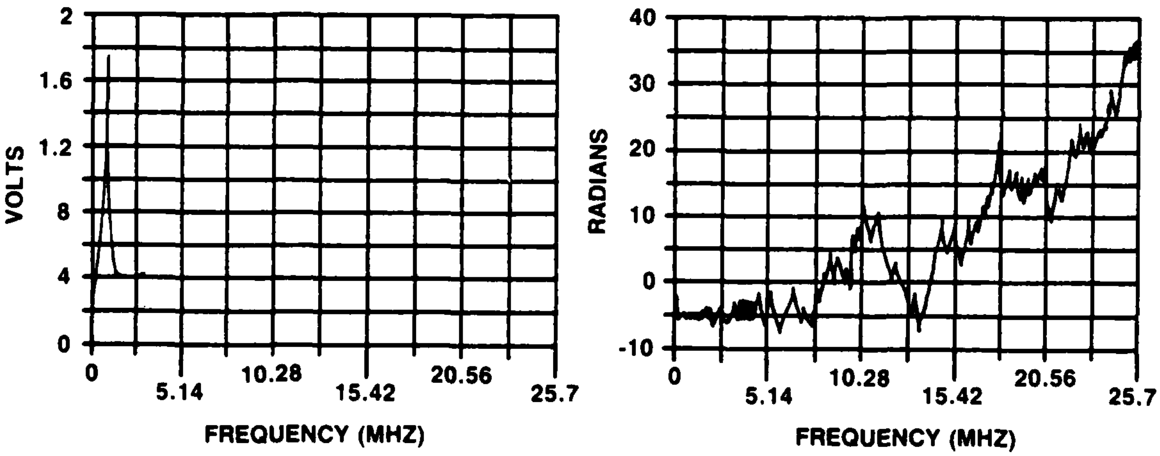

Figure 14. Polar Outputs

The polar outputs are further converted to decibels and degrees using standard conversions. The first 300 kHz portion of the spectra is shown in figures 15 and 16. A partial listing of the tabular output from which these spectra came is given in table 1.

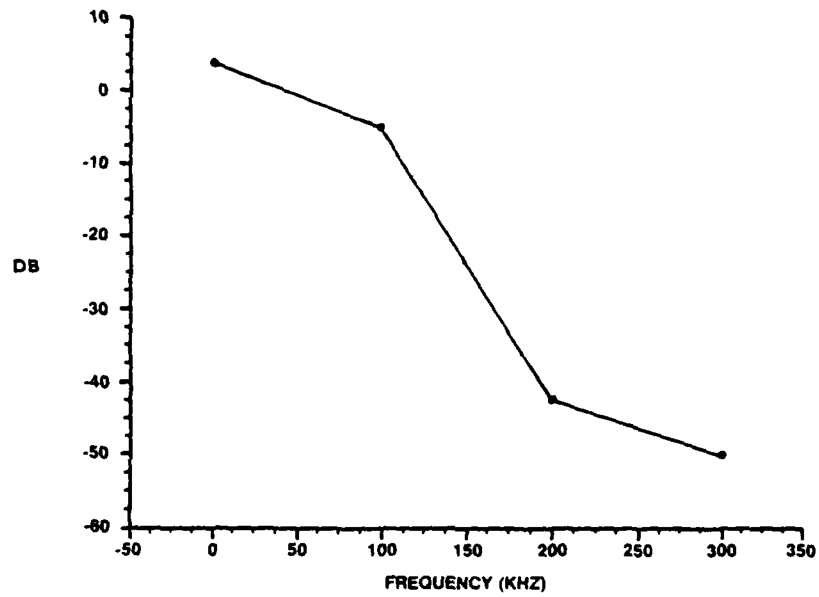

Figure 15. Expansion of First Four Polar Amplitude Points

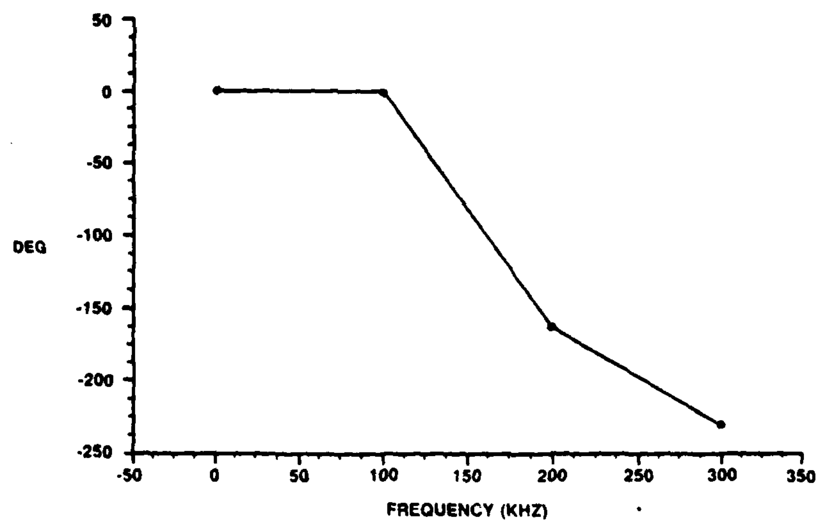

Figure 16. Expansion of First Four Polar Phase Points

Table 1. Partial Output of Polar Converted Values

Frequency	dB	Phase
0	4.15933	0
100000	-5.53384	4.76484
200000	-41.9041	-152.36
300000	-50.3202	-233.442
400000	-54.6881	-229.514
500000	-55.7992	-241.12
600000	-60.3684	-238.422
700000	-60.4694	-254.437
800000	-65.3596	-235.308
900000	-61.6856	-226.762
1.00000E+06	-64.4957	-255.243
1.10000E+06	-65.561	-262.515
1.20000E+06	-67.6994	-272.461
1.30000E+06	-67.5211	-232.314
1.40000E+06	-66.406	-252.113
1.50000E+06	-66.4135	-246.955
1.60000E+06	-68.8308	-272.633
1.70000E+06	-66.189	-244.664
1.80000E+06	-72.809	-303.204
1.90000E+06	-79.2125	-198.893
2.00000E+06	-67.8521	-211.732

Brief Review of FFT Input/Output Relationships

Before going on to other aspects of the FFT, it may be advantageous to review the facts that have already been encountered:

1. Frequency Increment = $\frac{\text{Sampling Frequency}}{\text{No. of Digitized Points}}$, or $FI = \frac{SF}{N}$.

2. The time window must be long enough to capture at least one whole cycle; i.e., $T = 1/F$, where T is time (seconds), and F is frequency (Hz).

3. Time/Digitized Points = T/N.

4. $SF = \frac{1}{\text{Time/Digitized Point}}$.

5. Highest Frequency of Spectra = SF/2 (known as the Nyquist frequency).

6. Number of Cycles in Measurement Window (NHz) = $\frac{(FI)\ (N)}{\text{Measured Frequency}}$.

7. Time Window (T) = N/SF.

8. The FFT outputs positive and negative frequencies. Only one-half of the number of time-domain input points are shown, which is the positive part of the frequency spectrum. The negative frequencies are ignored.

9. Note that the amplitude of table 1 corresponds to the peak value of figure 10. A simple correction would normalize this to an rms output. In this case, the FFT actually returns a level that is proportional to the peak energy content of each spectral line.

10. The phase value will be treated separately in a later section.

FREQUENCY RESOLUTION AND APPENDING ZEROS

Spectral Resolution

Although the FFT always outputs one-half the number of digitized points as spectral lines, the number of cycles in the time window affects the frequency resolution drastically. This is best shown by looking at actual FFT outputs. Figures 17 to 21 are all FFT outputs from a 100 kHz sine wave digitized into 512 points (N). For clarity, only the first 300 kHz amplitude values are shown. Keep in mind that there are actually 256 spectral lines.

Figure 17 is the result of one cycle in the time window. Figures 18, 19, and 20 are the result of 2, 5, and 10 cycles, respectively, in the time window. Slight differences in the amplitude of the various graphs are due to noisy signal conditions. This does not affect the spectrum resolution.

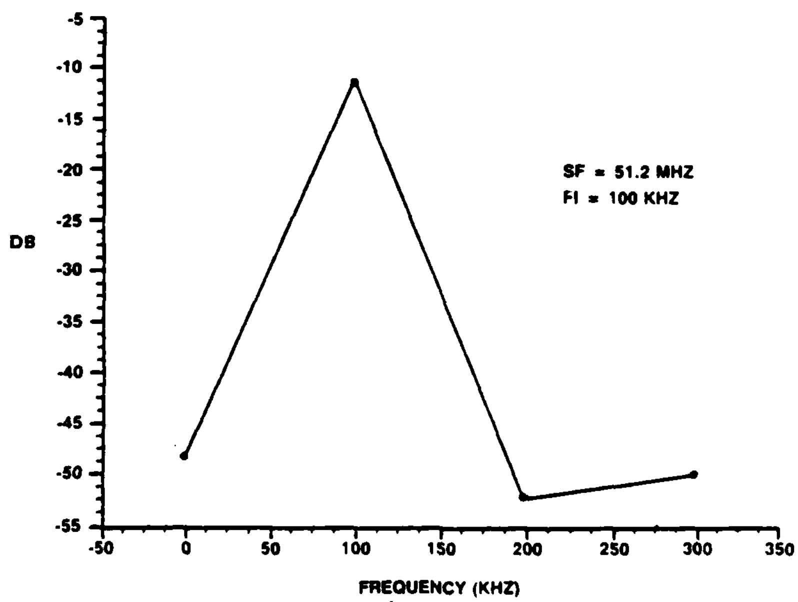

Figure 17. Spectral Output with One Cycle in Time Window

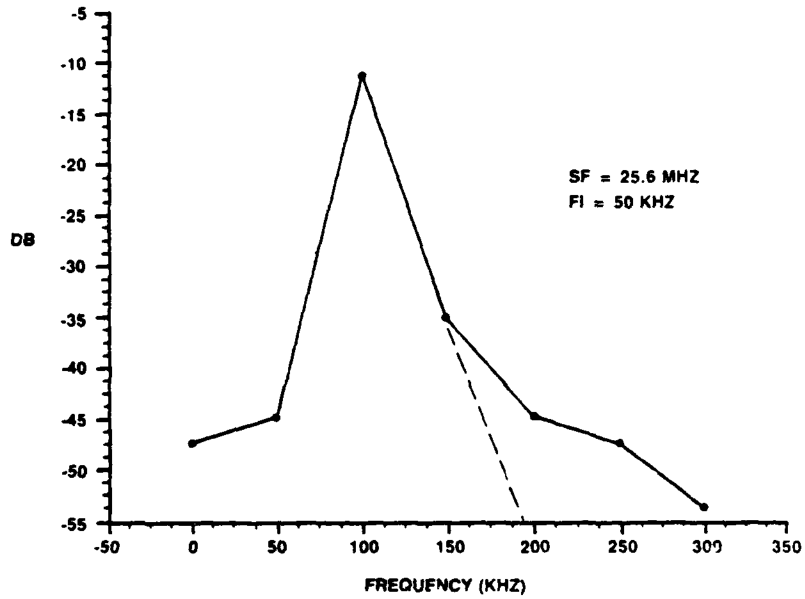

Figure 18. Spectral Output with Two Cycles in Time Window

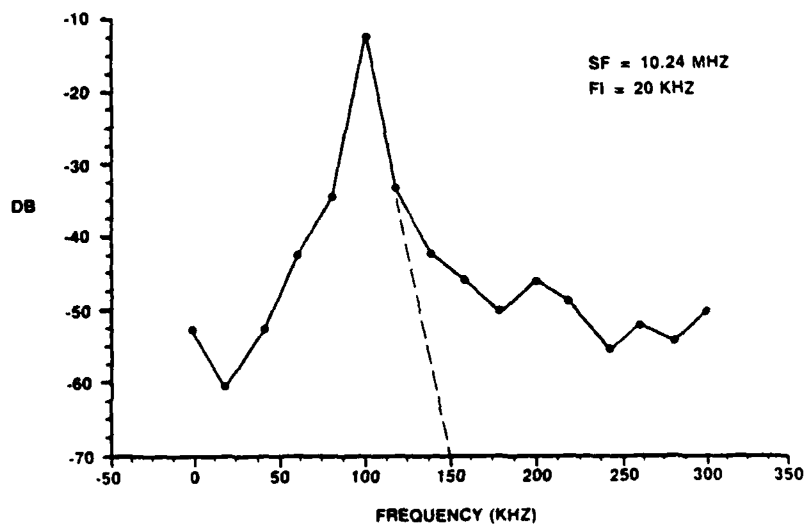

Figure 19. Spectral Output with Five Cycles in Time Window

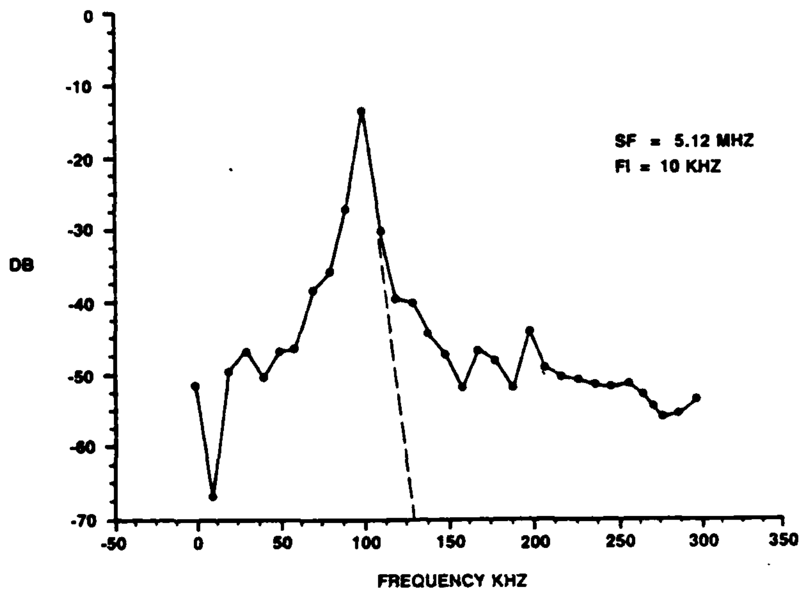

Figure 20. Spectral Output with Ten Cycles in Time Window

Zero Appending

Figure 18 shows that having two cycles in the time window gives twice the spectral resolution obtained with one cycle in the window. Figure 21 shows that spectral resolution can be increased by another method, called "appending zeros." This means that, if one has a 512-point time-domain window, the FFT spectral resolution can be doubled by appending another 512-point window to the first one. The appended window contains 512 zeros. If two 512-point windows are appended, resolution will be increased three times, just as if there were three cycles in the window. This is demonstrated in figure 21, which is the output of an FFT of a 512-point 1-cycle input, to which a 512-point window of zeros has been appended before the FFT is calculated.

It is important to note that the addition of points to the window increases the execution time of the FFT computation in a logarithmic ratio.

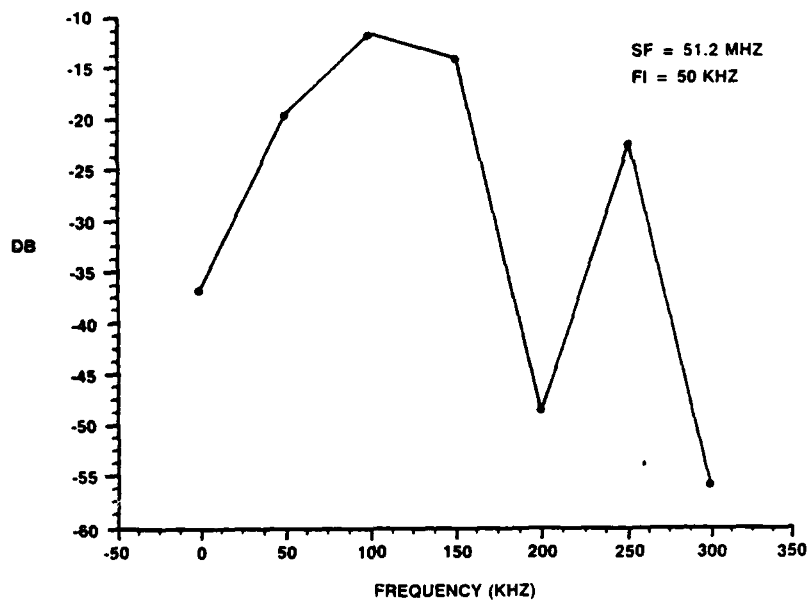

Figure 21. Spectral Output with One Window of Zeros Appended

Comments on Spectral Resolution

1. No. of Spectral Lines = N/2.

2. Lowest Frequency = dc (0 Hz).

3. First Frequency = First Increment = 1/T.

4. Highest Frequency = N/2T (Nyquist frequency).

5. (N/2) lines are spaced by (1/time record).

6. Short time record gives wide line spacing.

7. Long time record gives narrow line spacing.

8. Number of cycles in time window determines frequency resolution.
   Example: 100 cycles = 1 percent frequency resolution, or
   Resolution = (1/ No. of Cycles in Window)

9. Resolution can be increased by appending zeros while maintaining a short "gate" measurement time.

10. The FFT outputs the true peak energy amplitude of each frequency increment.

11. The frequency increment is inversely proportional to the number of points sampled (figure 22):

Figure 22. Frequency Increment Versus Number of Sampled Points

12. The number of cycles in the time window determines how many frequency increments will occur before the spectral line of the frequency being measured occurs. An example for a 100 kHz input follows:

No. of Cycles in Window	100 kHz @ Line Number
1	1
2	2
5	5
10	10

13. It is also important to note that appending zeros affects the polar amplitude output. The addition of an appended window will cause the polar conversion to output a level that is one-half the peak value of the measured waveform, while the absence of an appended window causes the conversion to output the peak amplitude of the measured waveform. Appending more windows affects the amplitude accordingly.

FFT PHASE OUTPUT

Phase From Polar Conversion

The FFT computes the phase from the ratio of the imaginary-to-real parts of the frequency domain. (Phase is valid only at points where magnitude exists.)

A phase reading of 0° (zero delay) would be a cosine wave that starts at the time zero point in the digitizer window.

These two statements apply to each and every spectral line whether or not they are background noise. (The phase of noise frequencies will not be valid.)

The phase spectrum expresses the phase shift of the frequency components relative to time zero (the first sample point). Phase is only a position

indicator, not an energy indicator. The use of sinusoids for description is continued in figure 23 to show examples of phase readings for various waveforms in the time window. The number of cycles in the window does not change the phase reading, since the phase value is determined at time zero.

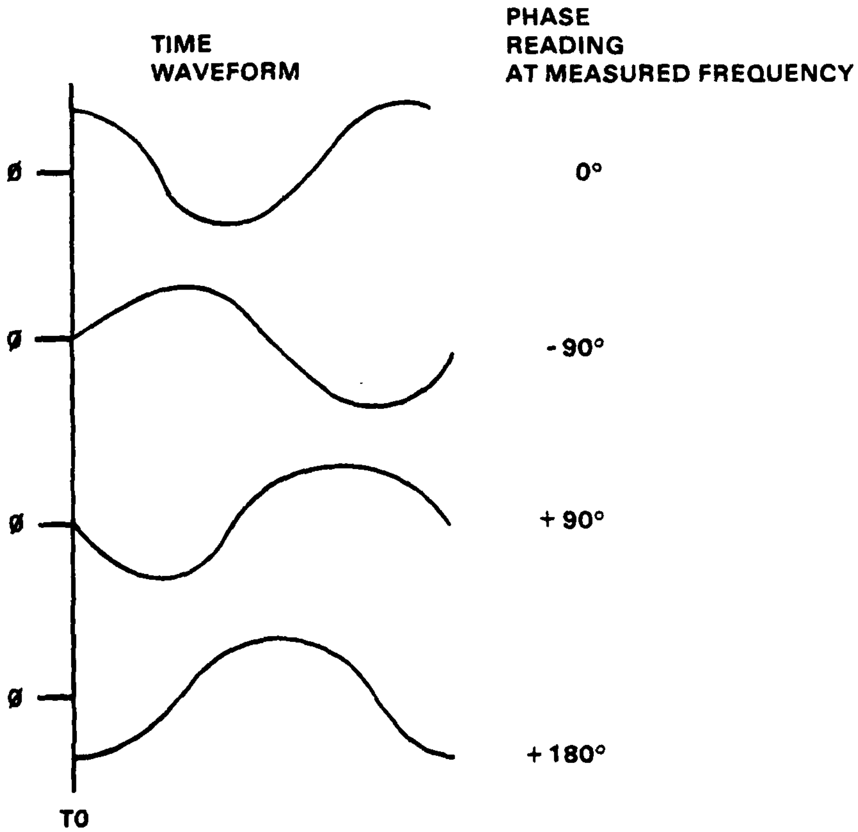

Figure 23. Phase Readings of Polar Output Versus Waveform Position in Time Window

Phase from Polar Conversion with Delay Function Applied

Delay Explanation. The phase spectrum expresses the phase shift of the frequency components relative to time zero (the first sample point) with the phase value limited to the range of $\pm\pi$ radians.

The delay of an individual signal is measured relative to time zero. The relative delay between two signals is then just the difference between the individual signal delays. Delay contributions that do not appear in the sampled signals, due to nonsimultaneous sampling, for example, do not contribute to the relative delay. A discontinuous phase display that ranges between $\pm\pi$ is called a modulo 2π presentation.

Each spectral output of the FFT is of a different frequency and therefore of different time periods. But since each period is time-shifted by the same amount (see figure 24), it follows that phase can be expressed as the ratio of time shift to the component's period:

$$\text{Phase} = -360^\circ \ \text{Shift/Period}.$$

Continuous phase shows the effects of time shifting (+ or -) directly. As an example, -360° phase is represented as $+360^\circ$ phase. Whereas modulo 2π phase views time shifting as a sinusoid, a cosine of zero phase appears identical to one shifted 360°. This means that all phases of 360° and 720° show up as 0° in modulo 2π phase. The same result applies to all phases in excess of $\pm\pi$ radians since they show no difference from a sinusoid around time zero.

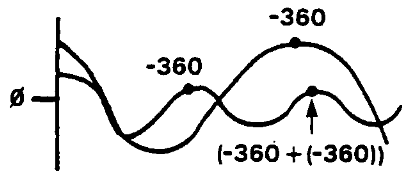

Figure 24. Phase Readout Versus Signal Time Period

Setting the delay parameter during the polar conversion provides continuous phase output. One approach to estimating delay between two signals is to estimate the frequency response function that relates the two signals. The delay is then estimated from the slope of the phase function. Signal delay shows up as a linear phase component that can be estimated and removed from the phase spectrum.

The relationship between delay and phase slope is:

$$\text{Delay} = -(\text{Slope})/2\pi,$$

where slope is expressed in radians/Hz and delay is in seconds. When delay is set prior to execution of the polar command, it causes the phase function to be "unwrapped" modulo 2π so that a continuous rather than discontinuous phase function results.

Use of the Delay Option. Delay is not applicable to narrowband signals such as sinusoids. It is sometimes needed when broadband signals such as shock pulses are used to extrapolate frequency responses from single-pulse excitations.

Examples of Delay Usage. Although delay slopes are somewhat meaningless on sinusoids, performing a zero delay during the polar execution does affect the output of the phase function. This is shown in figure 25.

A review of the various output values shows that the waveforms that have negative values at time zero have negative phase angles, while the opposite is

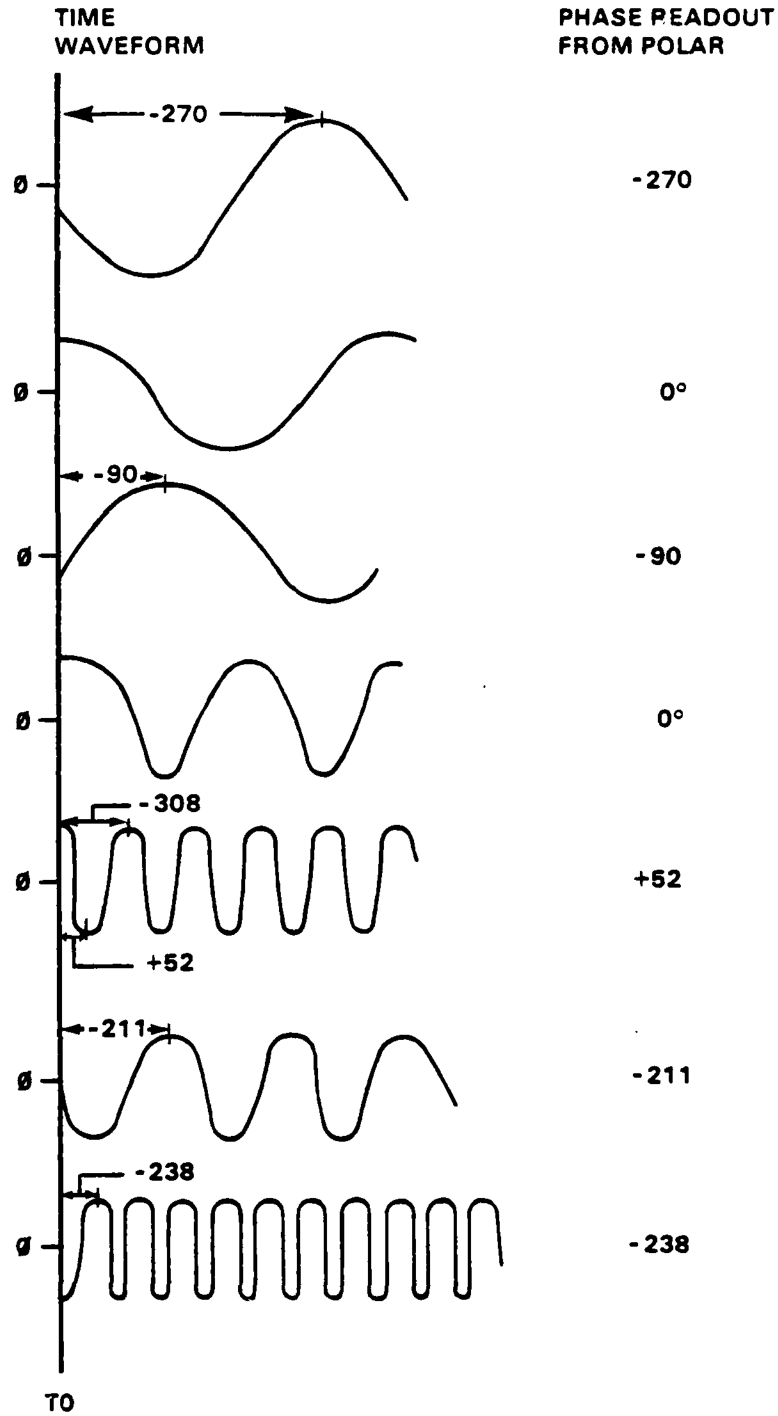

Figure 25. Phase Readout of Polar Output with Zero Delay Versus Waveform Position in Time Window

true for the waveforms that are positive at time zero. The values shown in figure 23 are the reverse since delay was not used.

Examples of the polar phase outputs are shown for delays of 0 and 10 in figure 26. Note the slope of the phase function when a delay greater than 0 is used. The outputs are for a 100 kHz cosine waveform.

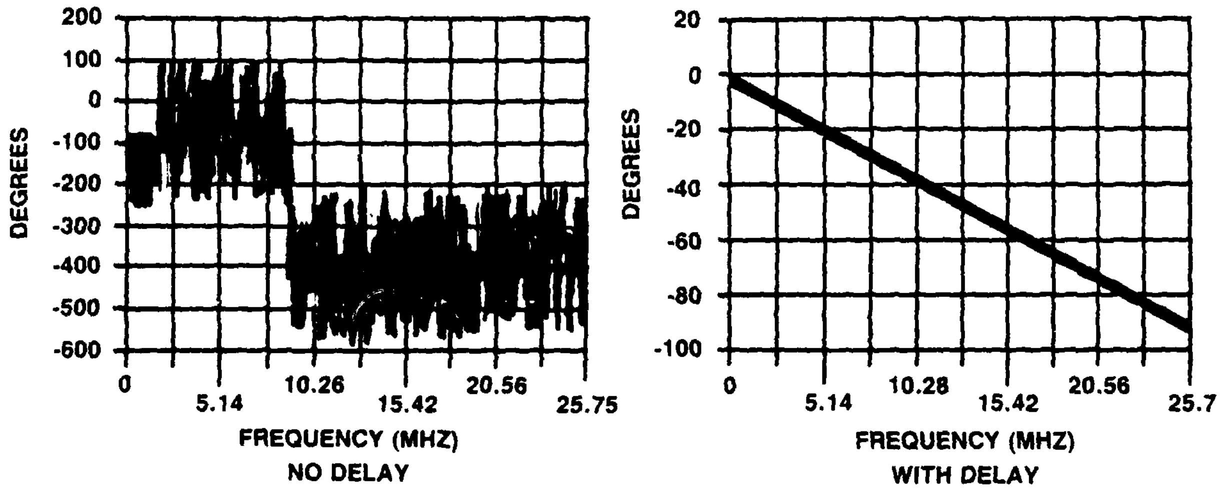

Figure 26. Effect of Delay on Phase Readout

WINDOWING

Nonperiodicity and Leakage

There are many instances when a signal in the time window is not completely periodic. The time window may not contain an integral number of sinusoids, or a pulse may not begin and end at the same voltage level. Figure 27 shows some examples.

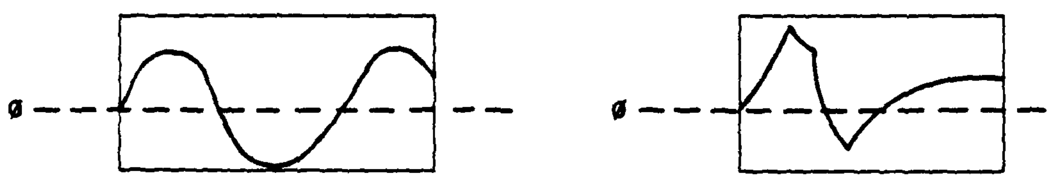

Figure 27. Examples of Nonperiodicity

This nonperiodicity causes "leakage" errors in the DFT output, which have the effect of "smearing" power across frequency components, as shown in figure 28.

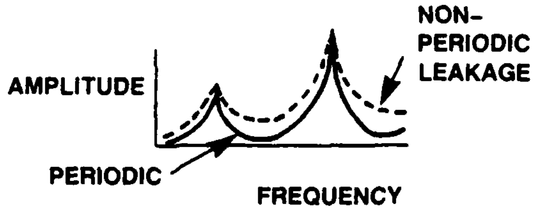

Figure 28. Effects of Leakage in DFT Output

Nonperiodicity of Sine Waves

There are three methods of ensuring periodicity when digitizing sine waves:

1. By adjusting window timing and sampling frequency so that an integral number of cycles is within the time window. An integral number of cycles in the window causes the waveform to be harmonically related to the window. Therefore, its frequency falls exactly on a sample point, which eliminates leakage.

2. If the digitizer does not permit the completion of an integral number of cycles, then the process can be finished in the software, by using interpolation techniques, prior to executing the FFT.

3. By applying a time window function that drives the end points of the digitized window to zero. This is shown in figure 29.

There are numerous time windowing functions. Each one has characteristics that make it more suitable for some situations than for others. A good general purpose window is the cosine tapered window. Here, various cosine taper values are assigned as percentages and serve to drive the nonperiodic signal to zero at the ends of the time window.

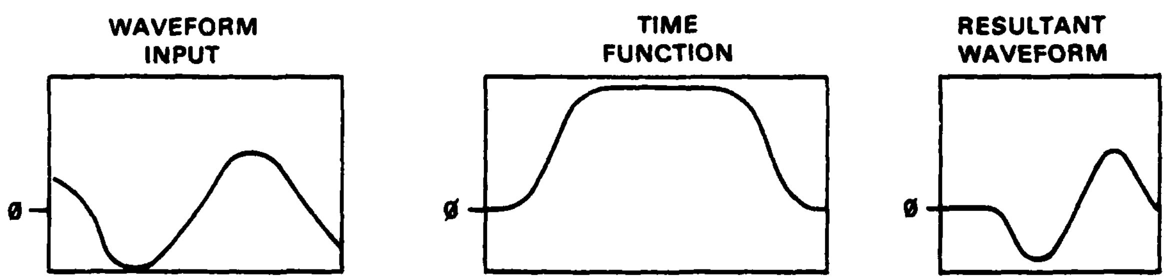

Figure 29. Effect of Applying Window to Ensure Periodicity

Nonperiodicity of Pulses

If pulses are nonperiodic, the only method of achieving periodicity is to use a time window function.

General Comments on Windowing

1. Keep in mind that time shifts in the window are expressed as delays. Therefore, the phase function will be affected by these shifts; the amplitude function will not be affected.

2. Windowing decreases the signal area and could affect the frequency-domain magnitudes. This can be compensated for by applying an inverse of the time window to the frequency spectra.

3. Waveforms with closely adjacent frequency components of nearly equal magnitudes are best sampled using the rectangular window. Because the major lobe width is not increased when this window is used, adjacent components do not tend to leak into each other.

4. When small spectral components are next to large ones, then one of the low sidelobe windows will make the small components more visible.

5. All time window functions have their own spectral characteristics. These characteristics are essentially multiplied with the waveform being sampled and, therefore, affect the final results. Table 2 shows the spectral content of the various windows that can be used to ensure periodicity. The table also indicates the effects of the time window on the signals being measured. In general, lower sidelobes mean less leakage, but more energy is concentrated into widening of the major lobes. Also, the major lobe magnitude varies with window shapes. This affects the magnitude spectra accordingly, and they must be corrected to compensate for this effect.

ALIASING

Cause and Effect of Aliasing

When the digitizer sampling frequency is less than twice the frequency being measured, aliasing occurs. This is called the Nyquist frequency:

FNyquist = Sampling Frequency/2.

The aliasing process causes the higher frequency components to be "folded over" into the spectrum of nonaliased frequencies:

Alias Frequency = Measured Frequency - Sampling Frequency.

Table 2. Some Common Data Windows and Their Frequency-Domain Parameters*

Unity Amplitude Window	Shape Equation	Frequency Domain Magnitude	Major Lobe Height	Highest Side Lobe (dB)	Band-width (3 dB)	Theoretical Roll-Off (dB/Octave)
Rectangle $T=1/\beta$	$A=1$ for $t=0$ to T	T	T	−13.2	0.86β	6
Extended Cosine Bell	$A=0.5(1-\cos 2\pi 5t/T)$ for $t=0$ to $T/10$ and $t=9T/10$ to T; $A=1$ for $t=T/10$ to $9T/10$		0.9 T	−13.5	0.95β	18 (beyond 5β)
Half Cycle Sine	$A=\sin 2\pi 0.5t/T$ for $t=0$ to T		0.64 T	−22.4	1.15β	12
Triangle	$A=2t/T$ for $t=0$ to $T/2$; $A=-2t/T+2$ for $t=T/2$ to T		0.5 T	−26.7	1.27β	12
Cosine (Hanning)	$A=0.5(1-\cos 2\pi t/T)$ for $t=0$ to T		0.5 T	−31.5	1.39β	18
Half Cycle Sine^3	$A=\sin^3 2\pi 0.5t/T$ for $t=0$ to T		0.42 T	−39.3	1.61β	24
Hamming	$A=0.08+0.46(1-\cos 2\pi t/T)$ for $t=0$ to T		0.54 T	−41.9	1.28β	6 (Beyond 5β)
Cosine^4	$A=(0.5(1-\cos 2\pi t/T))^2$ for $t=0$ to T		0.36 T	−46.9	1.79β	30
Parzen	$A=1-6(2t/T-1)^2-6\|2t/T-1\|^3$ for $t=T/4$ to $3T/4$; $A=2(1-\|2t/T-1\|)^3$ for $t=0$ to $T/4$ and $t=3T/4$ to T		0.37 T	−53.2	1.81β	24

*From R. Ramirez, "The FFT Fundamentals and Concepts," Tektronix Inc., 1975.

Elimination of Aliasing Effects

There are three methods of eliminating aliasing effects:

1. Ensure that the sampling frequency is at least twice that of the highest frequency to be measured.

2. Remove the higher frequencies from the signal with an antialiasing (lowpass) filter. The filter slope should be approximately 60 dB/octave.

3. There is a technique of working with aliasing called "undersampling." Since this method complicates the FFT output, further explanation is left to more technical publications.

DIGITAL FILTERING

Filtering is achieved by performing the FFT, eliminating the unwanted frequencies in the software, then reconstructing the waveform by performing an inverse FFT.

MEASURING WITH THE FFT

SINE WAVES

The following conditions must be considered when contemplating the use of the FFT for sine wave measurements of amplitude and phase.

1. Ensure that aliasing of the highest frequency component does not occur by adjusting the sampling frequency to be at least twice that of the frequency component. Actually four samples per measured cycle ensures better accuracy. (See above discussion of aliasing.)

2. Sinusoids are essentially of single frequency energy distribution. If the spectrum of a transfer function is wanted, enough frequencies must be alternately generated and measured (swept) to adequately define the function. The energy is centered at $F = \underline{1}/T$.

3. When distortion components are not needed, one cycle in the time window is adequate to accurately resolve amplitude and phase of the fundamental measured frequency. If distortion components are wanted (harmonics), enough cycles must be included in the time window to provide the required frequency resolutions. Refer to figure 18 which shows that having two cycles in the time window gives approximately 60 dB nulls between the harmonics; this is adequate. If "gate" width is not jeopardized, then three cycles in the window would be safer. The FFT is a filter at each spectral line whose bandwidth is affected by the number of cycles in the window.

4. Ensure periodicity. Refer to the earlier discussion of nonperiodicity of sine waves/pulses and methods for achieving periodicity.

5. Coherent digital time averaging may be required prior to waveform acquisition if the signal is noisy. Averaging gives a 3 dB noise improvement for each power of 2: (For example, 128 averaging = 2^7 = 7 x 3 dB = 21 dB.) Averaging also reduces time jitter and quantitizing errors. Frequency spectrum averaging can also be used when noncoherence prevents the use of time averaging.

6. Subtraction of the dc, 0-volt level is usually not necessary when measuring sinusoids. This would be done in the software prior to doing the FFT.

7. Accuracy of the final results is dependent on the linearity, bandwidth, and accuracy of the analog circuitry as well as the digitizer. The software accuracy is also to be considered. The data processing should be floating point and double precision if possible. The computer word size also affects the FFT accuracy since it determines the precision of the calculations. A good check on overall accuracy is to perform an inverse FFT of the frequency and phase spectra and reconstruct the original time waveform.

8. Ensure windowing requirements. (Refer to the earlier discussion of this subject.)

9. The window size is dependent on the digitizer memory size. The smaller memory will allow a minimum "gate" (measurement) time. But sampling frequency must be adequate. In general terms, a digitizer with a maximum sampling frequency of 20 MHz should adequately resolve a measured frequency of 3 MHz up to the third harmonic. With a time window of 64 points, this would give 9 cycles in the time window at this measured frequency, with a time of 3.2 microseconds; i.e.,

$$\text{Time/Pt} \times 64 = (5 \times 10^{-7})(64) = 3.2 \text{ microseconds.}$$

SHOCK PULSES

When using the FFT method to measure shock pulses, a complete transfer function is extrapolated from a single measurement of the shock stimulation and the system output due to that excitation. The following factors must be considered when contemplating use of the FFT for this measurement:

1. Aliasing must be guarded against. The user must ensure that all of the pulse transients are digitized adequately, or the FFT accuracy will suffer. To properly obtain accurate FFT spectra, the digitizer should obtain at least 10 points on the sharp transitions of the shock pulses. (Refer to the earlier discussion of aliasing.)

2. Digital averaging may be required as in sinusoid measurements.

3. It is necessary to subtract the dc, 0-volt level prior to executing the FFT.

4. Proper selection of time windowing most likely will be required in this case. (See the discussion of time windowing.)

5. A review of the factors outlined in sine wave usage may be helpful.

6. It is important to note that shock excitation will output the transient as well as steady-state responses of the system being tested. This may be detrimental when evaluating "high Q" systems.

SAMPLE PROGRAM TO DEMONSTRATE FFT OPERATIONS

Figure 30 is a flowchart of a program that demonstrates step-by-step operation of the FFT. The program listing, with explanations included, is provided in the appendix.

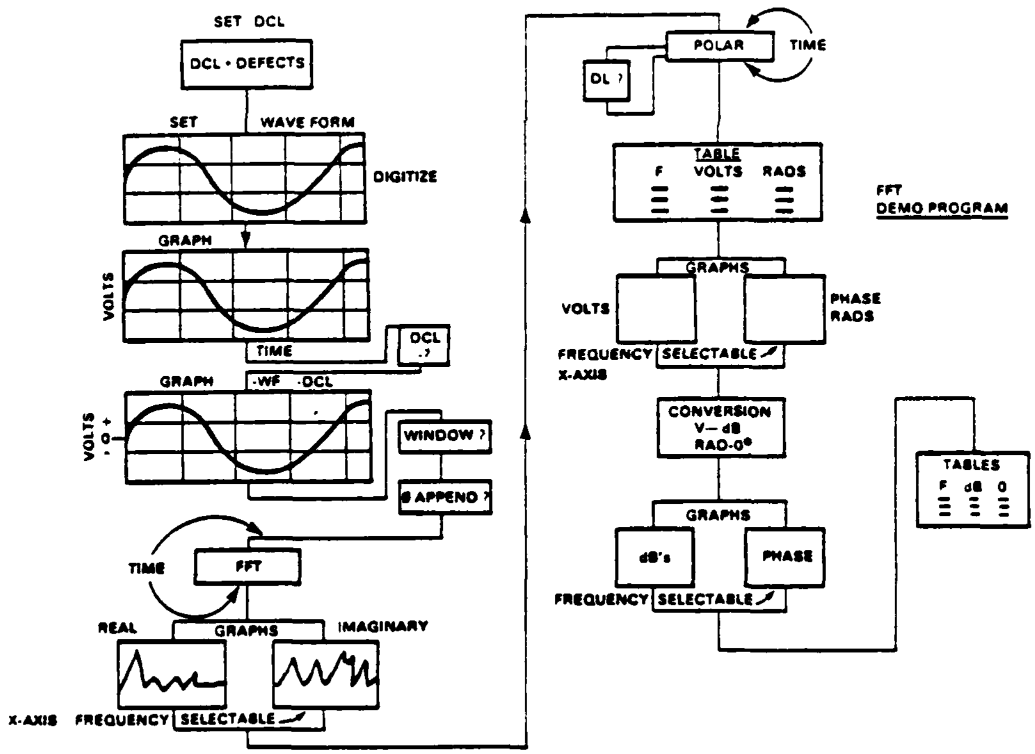

Figure 30. Flowchart of FFT Demonstration Program

USING THE FFT WITH NOISE SIGNALS

The FFT can be used with random or pseudorandom noise signals. Windowing is necessary when this method is used with random noise signals to evaluate nonlinear networks (amplitude dependent). Pseudorandom noise must be averaged, needs no windowing, and can be used on linear networks.

DIGITIZER SAMPLING FREQUENCIES

Table 3 was derived for a digitizer with a maximum rate of 20 MHz and window (memory) segmentation down to 64 points. The table is for a 64-point time window. The table was derived from the following formulas:

1. $SF = (N)(OP)$,

where OP is the measured frequency. When more than one cycle is wanted in the time window,

2. $SF = \frac{(N)(OP)}{X}$,

where X is the number of cycles in the window.

Table 3. Measured Frequency Versus Sampling Frequency

Measured Frequency (OP)	Sampling Frequency (SF)	Points/ Cycle	Cycles in Window	Frequency Increment (FI)	Approx. Time Window	Remarks
10 Hz	640 Hz	64	1	10 Hz	0.1 sec	Zero appending can be used to increase spectral resolution
100 Hz	6.4 kHz	64	1	100 Hz	0.01 sec	
1 kHz	64 kHz	64	1	1 kHz	0.001 sec	
10 kHz	640 kHz	64	1	10 kHz	0.1 msec	
100 kHz	6.4 MHz	64	1	100 kHz	0.01 msec	
312.51 kHz	>20 MHz 10.00032 MHz	32	2	156.255 kHz	6.3 .sec	
*						
625 kHz	13.333333 MHz	21.3	3	208.3 kHz	4.8 μsec	Zero appending not required
937.5 kHz	15. MHz	16	4	234.375 kHz	4.2 μsec	
1.25 MHz	16 MHz	12.8	5	0.25 MHz	4.0 μsec	
1.5625 MHz	16.666667 MHz	10.7	6	0.260417 MHz	3.8 μsec	
1.875 MHz	17.142857 MHz	9.1	7	0.267857 MHz	3.7 μsec	
2.1875 MHz	17.5 MHz	8	8	0.273438 MHz	3.6 μsec	
2.5 MHz	17.777778 MHz	7.1	9	0.277778 MHz	3.6 μsec	
2.8125 MHz	18.0 MHz	6.4	10	0.28125 MHz	3.5 μsec	
3.125 MHz	18.181818 MHz	5.8	11	0.284091 MHz	3.5 μsec	

*At this breakpoint two cycles must be allowed in the time window because the maximum sampling frequency has been reached. Two cycles satisfy the need for an integer number of cycles that always returns a spectral line at the measured frequency. Formula 2 (on page 29) is used in this case. Only the breakpoints are shown in the remainder of the table.

APPENDIX
COMPUTER ROUTINE FOR FFT OPERATIONS

```
10  REM  ** FFTDEM **    12/10/82
20  REM
100 IF Y9=1 THEN 330
110 Y9=1
120 RELEASE ALL
130 LOAD 'VM','GPI'
140 SIFTO @0,6000
150 OVLOAD VM:'SUBS'
160 REM
170 REM ** INITIALIZE VARIABLES **
180 DIM HS$(9),HU$(3)
190 HS$(0)="1." HS$(1)="2."  HS$(2)="5." HS$(3)="10."
200 HS$(4)="20." HS$(5)="50." HS$(6)="100." HS$(7)="200." HS$(8)="500."
210 HU$(0)="E-3" HU$(1)="E-6" HU$(2)="E-9" HU$(3)="E-12"
220 DIM VD$(8)
230 VD$(0)="5.E+0"
240 VD$(1)="2.E+0"
250 VD$(2)="1.E+0"
260 VD$(3)="5.E-1"
270 VD$(4)="2.E-1"
280 VD$(5)="1.E-1"
290 VD$(6)="5.E-2"
300 VD$(7)="2.E-2"
310 VD$(8)="1.E-2"
320 REM
330 LA=32 TA=64 SA=96
340 LOAD VM:"PAGE","PRINT", "WAIT"
350 PAGE
360 GOSUB 11000
370 AA$="CPL AC;RIN HI;V/D  5.E-1"
380 D=1 GOSUB 7000
390 AA$="LEV 0;MOD NOR;CPL AC;SRC LIN;SLO POS;T/D  100.E-6"
400 D=2 GOSUB 7000
410 REM  GTL AND RESET TV MODE
420 GOSUB 13000
430 REM  CLEAR SERVICE REQUEST
440 DELETE T1 DIM T1(2)
450 LOAD VM:"POLL","PRINT"
460 GOSUB 6000 GOSUB 6000 GOSUB 6000
470 DELETE T1 RELEASE "PRINT","POLL"
480 REM
490 PRINT "DIGITIZING DEFECTS . . . WAIT"
500 LOAD VM: "WHEN","IGNORE"
510 VR=0
520 WHEN @0 HAS "SRQ" GOSUB 3500
530 REM
540 REM  DIGITIZE DEFECTS
550 AA$="DIG DEF,100;XYZ DEF,OPC"
560 GOSUB 7000
570 IF VR<>1 THEN 570
```

```
580 REM
590 REM  SET SOURCE TO EXTERNAL
600 AA$="SRC EXT"
610 D=2 GOSUB 7000
620 REM  PREPARE FOR GROUND REFERENCE
630 AA$="CPL GND"
640 D=1 GOSUB 7000
650 PRINT PRINT "POSITION SIGNAL FOR GROUND REFERENCE"
660 PRINT "(FOR FFT TEST, POSITION ZERO REF LESS THAN 64)"
670 GOSUB 13000
680 PRINT MS$ WAIT
690 AA$="DIG SA,1;XYZ SA;READ SA"
700 D=0 GOSUB 9005
710 ZR=RMS(A)
720 PRINT PRINT "ZERO REF VALUE=",ZR
730 LOAD VM:"INPUT"
740 PRINT "ZERO REFERENCE SATISFACTORY", INPUT AU$
750 IF SEG(AU$,1,1) <> "Y" THEN 650
760 AA$="CPL AC"
770 D=1 GOSUB 7000
780 REM  RESET TV MODE AND GOTO LOCAL
790 D=0 GOSUB 13000
800 PAGE
810 PRINT "    SET UP WAVEFORM"
820 PRINT "(HIT ANY KEY WHEN READY)
830 WAIT
840 REM  ** ACQUIRE DATA **
850 PUT "DIG DAT;DEF ON;ATC;READ ATC" INTO @0,32,96
860 GOSUB 9007
870 A=A/2
880 REM  ** WAVEFORM ASSOCIATION **
890 GOSUB 4000
900 REM
910 PAGE
920 GRAPH AA
930 WAIT
940 PAGE
950 PRINT "DC LEVEL SUBTRACTION"; INPUT AU$ AU$=SEG(AU$,1,1)
960 IF AU$<>'Y' THEN 1040
970 DC=ZR
980 ZR=ZR*VA/64
990 AA=AA-ZR
1000 PAGE
1010 GRAPH AA
1020 WAIT
1030 REM
1040 REM  FFT MODIFICATIONS  8/13/80
1050 REM
1060 PAGE PRINT "WINDOW ROUTINE"; INPUT AU$ AU$=SEG(AU$,1,1)
1070 IF AU$<>'Y' THEN 1210
1080 REM
1090 REM  ** WINDOW ROUTINE TO 10% COSINE TAPER THE WAVEFORM **
1100 T3=ITP(512*10/100+.5)
```

```
1110 T2=3.141593/T3
1120 FOR J=0 TO T3-1
1130 T4=.5*(1-COS(T2*J))
1140 BB(J)=T4*BB(J)
1150 BB(511-J)=T4*BB(511-J)
1160 NEXT J
1170 PAGE GRAPH AA WAIT
1180 REM
1190 REM  ** ZERO-APPEND ROUTINE **
1200 REM
1210 PAGE PRINT "ZERO APPEND"; INPUT AU$ AU$=SEG(AU$,1,1)
1220 SZ=512
1230 DELETE A2,A3,W2,W3
1240 IF AU$<>'Y' THEN 1340
1250 A1=BB
1260 DELETE BB DIM BB(2*SZ-1)
1270 BB(0:SZ-1)=A1
1280 DELETE A1
1290 SZ=SZ*2
1300 REM
1310 REM
1320 REM  FFT PROCESSING ROUTINE
1330 REM  SET-UP WAVEFORMS
1340 WAVEFORM W2 IS A2(SZ/2),SI,H2$,V2$
1350 WAVEFORM W3 IS A3(SZ/2),SI,H3$,V3$
1360 REM
1370 REM  ** RFFT **
1375 REM
1380 RFFT AA,W2,W3
1400 REM
1520 REM  ** CALCULATE FREQUENCY ARRAY **
1530 DELETE F DIM F(SZ/2)
1540 FOR J=0 TO SZ/2
1550 F(J)=J*SI
1560 NEXT J
1563 REM
1565 GOSUB 4510
1570 REM
1580 REM  ** POLAR **
1585 REM
1590 PAGE PRINT "DO YOU A DELAY FOR POLAR"; INPUT AU$ AU$=SEG(AU$,1,1)
1600 PRINT PRINT "ENTER DELAY"; INPUT DL
1610 POLAR W2,W3,DL
1620 GOTO 1660
1630 POLAR W2,W3
1640 REM
1650 REM  ** DISPLAY TABLE **
1660 PAGE
1670 PRINT "FREQUENCIES", "VOLTS", "RADIANS"
1680 FOR J=0 TO SZ/2
1690 PRINT F(J),A2(J),A3(J)
1700 NEXT J
1710 PRINT PRINT "** END OF TABLE **"
```

```
1720 WAIT
1730 REM
1740 REM  ** CORRECTION **
1750 A2=A2*SI*2
1753 REM
1755 REM  ** GRAPH VOLTS AND PHASE (IN RADS) **
1760 GOSUB 4510
1765 REM
1770 REM ** CONVERT FROM VOLTS TO DB **
1780 A2=20*LOG(A2)/LOG(10)
1790 V2$='DB'
1810 REM
1830 REM  CONVERT RADS TO DEGS
1840 W3=W3*180/3.14159
1850 V3$='DEG'
1860 REM  ** GRAPH DB AND PHASE (IN DEGREES) **
1870 GOSUB 4510
1880 REM
1890 REM  ** DISPLAY TABLE **
1900 PAGE
1910 PRINT "FREQUENCY","DB","PHASE"
1920 FOR J=0 TO SZ/2
1930 PRINT F(J),A2(J),A3(J)
1940 NEXT J
1950 PRINT PRINT "** END OF TABLE **"
2000 END
3450 REM
3490 REM  ** OPC SERVICE REQUEST ROUTINE **
3500 VR=1
3510 IGNORE ALL
3520 RELEASE 'IGNORE', 'WHEN'
3530 RETURN
3900 REM
3990 REM  ** WAVEFORM ASSOCIATION ROUTINE **
4000 DELETE AA,BB BB=A WAVEFORM AA IS BB,SI,HA$,VA$
4010 LA=32 TA=64 D=0
4020 HA$="S"
4030 AA$="HS1" GOSUB 8000
4040 SI=VAL(SEG(AA$,5,LEN(AA$)-1))/51.2
4050 VA$="V"
4060 AA$="VS1" GOSUB 8000
4070 VA=VAL(SEG(AA$,5,LEN(AA$)-1))
4080 BB=((BB)*VA)/64
4090 RETURN
4100 REM
4500 REM  ** GRAPH ROUTINE **
4510 PAGE
4520 VIEWPORT 100, 500, 450, 750
4530 SETGR VIEW
4540 GRAPH W2
4550 VIEWPORT 600, 1000, 450, 750
4560 SETGR VIEW
4570 GRAPH W3
```

```
4580 WAIT
4590 PAGE PRINT "FREQUENCY RANGE SELECT"; INPUT AU$ AU$=SEG(AU$,1,1)
4600 IF AU$<>'Y' THEN RETURN
4900 REM
5000 REM  ** FFT FREQ. RANGE QUESTION **
5010 PRINT 'INPUT FREQUENCY RANGE (IN KHZ)'; INPUT T6,T7
5020 IF T6>T7 THEN 5130
5030 REM  ROUND OFF, INTERNAL SOFTWARE ERROR,  .00001 ADDED TO COMPENSATE
5040 T6=T6*1000/SI T6=ITP(T6+.50001)
5050 T7=T7*1000/SI T7=ITP(T7+.50001)
5060 IF T6<0 THEN 5110
5070 IF T7>SZ/2 THEN 5120
5080 IF T7=T6 THEN 5140
5085 GOTO 5200
5090 REM
5100 REM  * ERROR MESSAGES *
5110 PRINT 'LOWER FREQ MUST BE 0 KHZ OR GREATER' PRINT GOTO 5010
5120 PRINT 'HIGHER FREQ MUST BE';256*SI/1000;' KHZ OR LESS' PRINT GOTO 5010
5130 PRINT 'INPUT ASCENDING RANGES ONLY' PRINT GOTO 5010
5140 PRINT 'INTERVAL TOO SMALL, INPUT A LARGER RANGE' PRINT GOTO 5010
5150 REM
5190 REM  ** GRAPH SELECTED FREQUENCY **
5200 PAGE XYPLOT F(T6:T7),A2(T6:T7)
5210 WAIT
5220 PAGE XYPLOT F(T6:T7),A3(T6:T7) WAIT
5230 PAGE PRINT "ANOTHER FREQ. RANGE"; INPUT AU$ AU$=SEG(AU$,1,1)
5240 IF AU$='Y' THEN 5010
5250 RETURN

READY
*
```

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